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Structure of an electronic–ionic gas

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Abstract. Using the idea of a local hydrodynamical equilibrium in a two-component Coulomb gas, one can prove that the latter possesses a spatially periodic structure. The spatial period calculated is found to be in good agreement with the period experimentally measured for gas plasma.

In the works of Dyson (1962) it is shown that the Coulomb gas is a mono-phase system, in which no phase transitions take place at any finite values of the temperature. Furthermore one obtains that the Coulomb gas possesses a long-range order structure at an arbitrary temperature, which is an analog to the crystal one. In the works of Martinov *et al.* (1969) the question of the existence of a spatially periodic structure in a two-component Coulomb gas has been considered by a hydrodynamical treatment of one-dimensional case.

The purpose of the present paper is to prove the existence of a periodic spatial structure in the three-dimensional Coulomb gas, placed in an external homogeneous dc field $\boldsymbol{E}$. The consideration is accomplished within the limits of the hydrodynamical approximation for describing the steady state of the system, using the idea of the mutual compensation of the field and statistic factors (Vlasov 1966). This idea expresses in fact the condition of a local hydrodynamical equilibrium. Therefore, one obtains for the system of input equations:

$$\begin{aligned}
&\frac{1}{\rho_1(\boldsymbol{\tau})}\nabla p_1(\boldsymbol{\tau}) + \nabla\{e\varphi(\boldsymbol{\tau})\} - m\nu(\boldsymbol{v}_2 - \boldsymbol{v}_1) = 0 \\
&\frac{1}{\rho_2(\boldsymbol{\tau})}\nabla p_2(\boldsymbol{\tau}) - \nabla\{e\varphi(\boldsymbol{\tau})\} - m\nu(\boldsymbol{v}_1 - \boldsymbol{v}_2) = 0 \\
&\Delta\varphi(\boldsymbol{\tau}) = -4\pi e\rho_1(\boldsymbol{\tau}) + 4\pi e\rho_2(\boldsymbol{\tau}) \\
&p_1(\boldsymbol{\tau}) = \theta\rho_1(\boldsymbol{\tau}) \qquad p_2(\boldsymbol{\tau}) = \theta\rho_2(\boldsymbol{\tau})
\end{aligned} \tag{1}$$

where the subscript 1 refers to the values characterizing the ionic component; the subscript 2 refers to the electronic component. The inertial forces are not taken into account in equation (1) inasmuch as only the steady state of the electronic–ionic gas ($\mathrm{d}\boldsymbol{v}/\mathrm{d}t = 0$) is considered, as mentioned above. $\rho(\boldsymbol{\tau})$ is the spatial distribution of the particles, $\varphi(\boldsymbol{\tau})$ is the selfconsistent potential, $p(\boldsymbol{\tau})$ is the pressure of the components, $\theta = \kappa T_1 = \kappa T_2$ where T is the absolute temperature, κ the Boltzmann constant, e the electronic charge; only singly charged positive ions are being considered. We assume that the external electric field is applied along the OX axis, so that $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, representing the steady-state drift velocities of the ions and the electrons, are: $\boldsymbol{v}_1(v_1, 0, 0)$, $\boldsymbol{v}_2(v_2, 0, 0)$. $|\boldsymbol{v}_2 - \boldsymbol{v}_1|$ is assumed to be constant which holds true for a fully ionized plasma in steady state, the latter being realized as a result of friction. It can be assumed, therefore, that the ion and electron velocities are constant both in space and time.

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The quantity ν is the effective frequency of the collisions of the electrons and the ions ($\nu = \nu_{ei} = \nu_{ie}$), inasmuch as the system represents a fully ionized plasma. In equation (1) m is the effective mass of the particles $m = m_1 m_2/(m_1+m_2 \simeq m_2)$.

Equations (1) are to be solved using the following boundary conditions:

$$\begin{aligned} \varphi(\boldsymbol{\tau} = 0) &= \varphi(0) \qquad & \rho_1(\boldsymbol{\tau} = 0) &= \rho_1(0) \\ \varphi(x = L, 0, 0) &= \varphi(L) \qquad & \rho_2(\boldsymbol{\tau} = 0) &= \rho_2(0) \end{aligned} \tag{2}$$

where L is the size of the system along the OX axis.

The external homogeneous dc field is determined by the potentials given in the following expression:

$$E = \frac{\varphi(L)-\varphi(0)}{L}.$$

From (1) one gets the following equations for $\varphi^*(\boldsymbol{\tau})$:

$$\Delta\varphi^*(\boldsymbol{\tau}) = \frac{4\pi e^2 \rho(0)}{\theta}[\exp\{\varphi^*(\boldsymbol{\tau})\} - \exp\{-\varphi^*(\boldsymbol{\tau})\}] \tag{3}$$

where

$$\varphi^*(\boldsymbol{\tau}) = \frac{1}{\theta}[m\nu(v_2-v_1)x - e\{\varphi(\boldsymbol{\tau})-\varphi(0)\}] + \chi.$$

$\rho(0)$ and x are determined by

$$\rho_1(0) = \rho(0)\exp(X) \qquad \rho_2(0) = \rho(0)\exp(-X)$$

so that

$$\rho(0) = \{\rho_1(0)\rho_2(0)\}^{1/2} \qquad \chi = \ln\left(\frac{\rho_1(0)}{\rho_2(0)}\right)^{1/2}.$$

Equation (3) permits a periodic solution for $\varphi^*(\boldsymbol{\tau})$ of the type

$$\varphi^*(\boldsymbol{\tau}) = \ln\tan^2\left(\frac{\pi}{D}(x+y+z) + \tan^{-1} e^{\chi/2}\right) \tag{4}$$

where

$$D = \pi\sqrt{3}\left(\frac{\theta}{2\pi e^2\rho(0)}\right)^{1/2}. \tag{5}$$

For the selfconsistent potential $\varphi(\boldsymbol{\tau})$ one obtains from (4)

$$\varphi(\boldsymbol{\tau}) = \frac{\theta}{e}\chi + \frac{m\nu}{e}(v_2-v_1)x - \frac{\theta}{e}\ln\tan^2\left(\frac{\pi}{D}(x+y+z) + \tan^{-1}e^{\chi/2}\right) + \varphi(0). \tag{6}$$

For the distributions $\rho_i(\boldsymbol{\tau})(i = 1, 2)$, from equation (1) it follows that

$$\begin{aligned} \rho_1(\boldsymbol{\tau}) &= \rho(0)\tan^2\left(\frac{\pi}{D}(x+y+z) + \tan^{-1}e^{\chi/2}\right) \\ \rho_2(\boldsymbol{\tau}) &= \rho(0)\cot^2\left(\frac{\pi}{D}(x+y+z) + \tan^{-1}e^{\chi/2}\right). \end{aligned} \tag{7}$$

In as much as classical electrodynamics is used, equations (6) and (7) possess singularities at certain values of $\boldsymbol{\tau}$. These singularities can be removed by introducing a finite size of the charge carriers, that is by performing the corresponding cut-off.

Besides the well-known homogeneous solutions for the densities of the particles and the selfconsistent potential in an electronic–ionic gas ($\rho_1(\mathbf{r}) = \rho_2(\mathbf{r}) = \text{const}$, $\varphi(\mathbf{r}) = \text{const}$), equations (6) and (7) indicate that the Coulomb gas can possess a periodic spatial structure with period D.

The following expression is a result of the satisfaction of the second boundary condition for the potential and the condition $L = lD (l = 1, 2, 3, \ldots)$, that is, when the size of the region of the system along the OX axis holds a whole number of characteristic spatial periods:

$$\frac{\varphi(L, y, z)-\varphi(0, y, z)}{L} = E = \frac{mv}{e}(v_2 - v_1). \tag{8}$$

This expression describes correctly the relative steady state drift of the particles $u = eE/mv$ with account of the collisions between the particles of the Coulomb gas.

Having in mind equation (1) it follows that, in the absence of an external electric field, the Coulomb gas possesses the same spatial periodical structure.

Therefore the results obtained indicate that every electronic–ionic gas can possess a spatial structure, by analogy with the crystal structure. The characteristic spatial period determined by us is $2\pi\sqrt{3}$ times greater than the respective Debye radius of screening.

The results obtained confirm unambiguously Dyson's statement (Dyson 1962) that every Coulomb gas must be called a 'crystal' rather than a 'gas'.

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